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ROCK FAILURE CRITERIA FOR DESIGN OF UNDERGROUND
STRUCTURAL SUPPORTS(U) ARMY ENGINEER WATERWAYS
EXPERIMENT STATION VICKSBURG MS GEOTECHNICAL LAB

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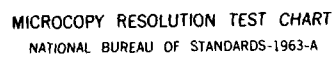
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ROCK FAILURE CRITERIA FOR DESIGN OF UNDERGROUND STRUCTURAL SUPPORTS

by

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20. ABSTRACT (Continued).

feasibility of developing a strain-based failure criterion for rock.

Stress-strain responses for two rock types, sandstone and granite, were evaluated. Reasonable correlations were found to exist between axial strain at failure normalized with respect to unconfined compressive axial strain at failure and the following:

- a. Maximum shear stress normalized with respect to the unconfined compressive strength.
- b. Minor principal stress (confining stress) normalized with respect to the unconfined compressive strength.

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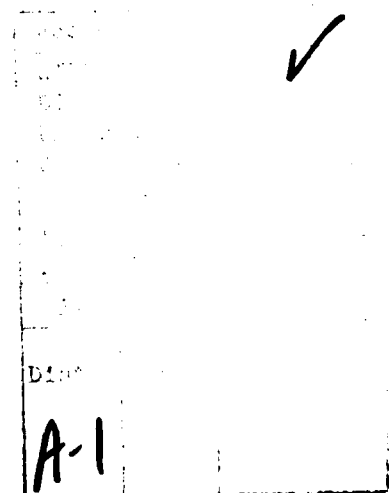
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PREFACE

The study reported herein was performed under Department of the Army Project No. 4A161101A91D, Task Area 02, Work Unit 157, entitled "Rock Failure Criteria for Design of Underground Structural Supports," In-House Laboratory Independent Research (ILIR) Program, sponsored by the Assistant Secretary of the Army (R&D).

The investigation was conducted by the US Army Engineer Waterways Experiment Station (WES) during the period Fiscal Year 1984-Fiscal Year 1985. The study was conducted under the direct supervision of Mr. J. S. Huie, Chief, Rock Mechanics Applications Group (RMAG), Engineering Geology and Rock Mechanics Division (EGRMD), Geotechnical Laboratory (GL), and under the general supervision of Dr. D. C. Banks, Chief, EGRMD, GL. Dr. W. F. Marcuson III was Chief, GL. Ms. M. E. Glynn, RMAG, assisted in the collection and reduction of test data. Ms. M. A. Kirklin, EGRMD, typed the initial draft of the report. Mr. G. A. Nicholson, RMAG, prepared this report.

COL Tilford C. Creel, CE, and COL Robert C. Lee, CE, were Commanders and Directors of WES during the conduct of the study. COL Allen F. Grum, USA, was Director of WES during the preparation and publication of this report. Mr. Fred R. Brown and Dr. Robert W. Whalin were Technical Directors.



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ROCK FAILURE CRITERIA FOR DESIGN OF UNDERGROUND STRUCTURAL SUPPORTS

PART I: INTRODUCTION

Background

Failure Criteria

Bieniawski (1984) defines a criterion of failure as "an algebraic expression of the mechanical condition under which a material fails by fracturing or deforming beyond some specified limit. This specification can be in terms of load, deformation, stress, strain, or other parameters." Bieniawski listed no less than fourteen failure criteria that have been used in the past or are currently being used to define failure for rock. These criteria are listed in Table 1. While the empirical rock mass strength criterion proposed by Hoek and Brown (1980) (see Table 1) attempts to define failure in rock masses, the majority of the criteria listed in Table 1 were either specifically developed for intact rock or adopted to define intact rock failure.

Table 1 was not intended to be all inclusive. In addition to the criteria listed, criteria are available for a third potential mode of failure - sliding along a single discontinuity. The better known of the single plane sliding failure criteria include the bilinear criterion proposed by Patton (1966) and Goldstein et al. (1966) and the curvilinear criterion proposed by Barton (1971 and 1973), and Ladanyi and Archambault (1969).

The mechanical complexity of failure increases from intact rock to discontinuous rock to rock mass. Although much progress has been made over the past 200 plus years since Coulomb proposed his criterion for failure in 1773, the practicing engineer is still in need of a general criterion that better defines rock failure. However, because of basic differences in mechanical behavior between the potential modes of rock failure, it is doubtful that a single theoretical criterion will ever be developed to describe all three modes of failure. Bieniawski (1984) is of the opinion that to meet the immediate needs of the practical rock engineer, attention should be directed to empirical criteria for estimating the triaxial strength of rock. Such criteria can be selected by fitting a suitable equation into experimental data and they need not have a theoretical basis.

Because of the complexities associated with rock failure, it is doubtful

that failure criteria founded on theoretical considerations will be forthcoming in the near future. In this respect, empirical criteria offer a viable interim approach for predicting stress-strain responses of deep underground excavations such as the Deep Basing concept being considered for deployment of the MX missile.

Existing Strain Based Criteria

An extensive literature review located only one failure criterion based on strain. The criterion was proposed by St. Verrant in 1870 (Nadai, 1950) and became known as the Maximum Elastic Strain Theory or the Equivalent Stress Theory. The criterion was developed for expressing the condition of brittle failure and even yield for metals. According to this theory, the maximum positive elastic extension of the material determines failure of either kind. From Hooke's Laws of Elasticity, if the elastic strain defined by Equation 1 is positive, (sign convention tension is positive)

$$\epsilon_1 = \frac{1}{E} [\sigma_1 - \nu (\sigma_2 + \sigma_3)] \leq \frac{\sigma_1}{E} \quad (1)$$

where ϵ_1 = Major principal strain

E = Modulus of elasticity

$\sigma_1, \sigma_2, \sigma_3$ = Major, intermediate, and minor principal stresses, respectively.

ν = Poisson's ratio

then the expression

$$\sigma_1 - \nu (\sigma_2 + \sigma_3) \leq \sigma_0 \quad (2)$$

should be smaller than the positive limiting "equivalent" (fracture or yield) stress σ_0 .

Although the Maximum Elastic Strain Theory enjoyed considerable attention, particularly in France and Germany, around the turn of the century, the theory as a failure criteria is not suited for rock. Consider the case of simple uniaxial compression (i.e. $\sigma_2 = \sigma_3 = 0, -\sigma_1$) where from Hooke's Laws the lateral strain is positive according to:

$$\epsilon_2 = \frac{1}{E} [-\nu(-\sigma_1)] = \frac{\nu \sigma_1}{E} \quad (3)$$

where ϵ_2 = Intermediate principal strain.

According to this criterion for uniaxial compression $\nu \sigma_1 \leq \sigma_0$ or $\sigma_1 \leq \frac{\sigma_0}{\nu}$ (from Equations 1 & 2 for pure tension $\sigma_1 \leq \sigma_0$). Hence it can be seen

that the equivalent fracture stress in compression is greater by a factor of σ_0/ν than the fracture stress in tension. Laboratory experience with rock indicates that a typical value for Poisson's ratio is on the order of 0.20. In such a case, the criterion would predict that the uniaxial compressive strength is 5 times greater than the tensile strength. However, laboratory test data indicate that the uniaxial compressive strength for most intact rock typically varies from 10 to 15 times the tensile strength.

The primary short comings of the Maximum Elastic Strain Theory, as applied to rock, is that the Theory assumes that the material behaves in a perfectly elastic manner. Rock, in particular a rock mass, can seldom be expected to behave as an elastic material.

Scope

This report briefly examines the role of constitutive stress-strain relationships as necessary input to numerical analysis of rock wall/support responses of underground excavations. Currently used models, as well as a proposed constitutive model, are discussed in terms of their applicability and limitations. The proposed constitutive model requires the development of a failure criterion that defines strain at failure. Toward this end, the feasibility of developing a strain based failure criterion for two intact rock types are investigated. The two rock types are intact sandstone and intact granite. Data for the two rock types were obtained from triaxial tests results published in the literature.

PART II: CONSTITUTIVE RELATIONSHIPS

Current Practice

For convenience of discussion, one can divide the design of structures constructed on or within a rock mass into two general categories: surface or near surface structures and underground excavations. The distinction between the two categories lie in the manner in which acting stresses are generated. Acting stresses include stresses tending to cause instability (driving stresses) and stresses acting normal to the potential failure surface (normal stresses). For surface or near surface structures (e.g. gravity dams and slopes) acting stresses are, as a rule, independent of strain. Resistance to the stresses acting on the structure is generated by the inherent strength of the rock mass or interface between the rock mass and the structure. While resisting stresses in such cases are dependent upon strain, satisfactory stability determinations can be obtained by ignoring this dependency upon strain and specifying in the stability analyses the appropriate upper limit of resisting stress (shear strength) that can be developed. Upper limits of resisting shear stress are usually defined by a failure criterion which relates either the principle stresses or the normal stresses to the shear strength that can be developed. Suitable factors of safety, based on experience, are specified for the solution of stability. Factors of safety primarily account for the uncertainty of the assigned upper limit of resisting shear strength. However, for structures that can tolerate only small deformations during operation, appropriate factors of safety also minimize the amount of strain necessary to develop the resisting stress required by the design.

In underground excavations, as the preexisting rock support is removed during excavation, displacements occur in the rock surrounding the opening, and stresses are redistributed. Hence stress acting within the wall rock and on the structural supports are, in part, a function of the relaxation (strain) process. Mechanisms of interaction between the rock walls and supports are complex. Because of the complexities the design processes for structural support rely almost entirely on empirical rock mass classification systems (i.e. Terzaghi (1946), Bieniawski (1973), Barton, Lien, and Lunde (1974) and others). This does not mean to imply that numerical approaches are not available for modeling rock wall/support responses.

Numerical approaches can be separated into two general groups - continuum

and discontinuum. Continuum approaches include the finite element, finite difference, and boundary element methods. Discontinuum models feature numerical approaches involving equation of motion of particles or blocks. Such models are commonly referred to as discrete element methods. A continuum approach treats the rock mass as a continuum intersected (in some models) by a number of discontinuities, while discontinuum approaches treat the rock mass as an assemblage of independent blocks. Discontinuum approaches are primarily used when analyzing the displacement of independent and recognizable rock blocks. Of the numerical approaches available, the finite element method is, by far, the most popular and perhaps the most potentially useful analytical design tool.

The finite element concept as applied to continuum mechanics was introduced by Turner et al. (1956) for the analysis of aircraft frames. The first practical application of the method to geotechnical problems known to the author was made by Clough (1962) for the Little Rock District, U. S. Army Corps of Engineers, in a study of Norfolk Dam. Since the early days of development, the finite element method has evolved to become quite sophisticated.

Numerical modeling of the relaxation process dictates the use of constitutive models which define stress-strain behavior. The U. S. National Committee for Rock Mechanics (1981) listed no less than 15 numerical computer codes suitable for application to underground excavations. The codes, considered representative of the state of the art, are capable of modeling one or more of the following constitutive relationships.

- 1) Linear elastic
- 2) Nonlinear elastic
- 3) Linear viscoelastic
- 4) Nonlinear viscoelastic
- 5) Elasto-plastic
- 6) Elasto-visco-plastic
- 7) Dilatancy

Brady and St. John (1982) are rightly of the opinion that much more model application and verification are required before any of the above constitutive models can be applied for practical design of underground excavations. The reason the method is not considered a reliable design tool at the present does not lie in the method's ability to model the complexities associated with the relaxation process but rather reflects the inability of the geotechnical engineer to provide realistic input parameters.

The difficulties of providing realistic input parameters can best be illustrated by considering the simple linear elasto-plastic constitutive model illustrated in Figure 1. Several alternatives are available for defining the model. A frequently used alternative specifies the deformation moduli, defined in Equation 4, as input parameters.

$$E_d = \sigma_{d_f} / \epsilon_f \quad (4)$$

where E_d = Deformation modulus
 σ_{d_f} = Deviator stress at failure
 ϵ_f = Axial strain at failure (major principal strain)

Deformation moduli define the initial slopes of the stress-strain lines. A suitable stress based failure criterion is then specified to define the upper limits of deviator stress at failure. A second alternative specifies both deviator stress and strain at failure.

Good approximations of input parameters for either alternative can be obtained from triaxial tests on small intact or broken rock specimens. However, because of scale effects, stress-strain responses of small test specimens seldom model prototype rock mass behavior. Hence, input parameters for practical underground excavation problems involve considerable speculation. Generally, in practical problems, a deformation modulus is estimated from in situ tests such as the plate bearing test, and the upper limits of deviator stress are defined by a failure criterion. Such an approach incorporates two rather severe restrictions into the analysis.

First, in situ tests available for estimating the deformation modulus for rock mass are typically limited to an applied stress range of 0 to 6.9 MPa (0 to 1000 psi). The maximum capacity of available loading systems generally limit the upper end stress range. Complete stress-strain responses for most rock mass require stress ranges on the order of one magnitude greater than maximum stresses available for in situ testing. In addition, stress-strain responses are typically strangely curvilinear at low-stress levels as illustrated in Figure 2. The strong initial curvature is primarily due to closure of discontinuities (joints) within the rock mass and to some extent closure of microcracks within the intact segments of rock. The number, orientation, tightness, and type of infilling of the joints control the extent of curvature. Modulus values determined from in situ tests at relatively low stress levels, for example the slope of line OA in Figure 2.

are dilatant at stress levels approaching failure, at least for brittle failure. Hence, one would expect lower peak deviator stresses from drained tests than from undrained tests. The fact that this trend is not evident in Figure 6 suggests that the drained tests specimens might not have been completely saturated or that reduction in deviator stress was insufficient to generate a noticeable trend within the overall scatter of data.

Brittle-ductile transition. Insufficient detailed data prohibit a conclusive distinction between brittle and ductile failure. However, the limited data available in the form of stress-strain curves suggest that the majority of sandstones included in this study behaved in a brittle manner for normalized maximum shear stress ratios below approximately 3.5. This maximum shear stress ratio corresponds to a normalized axial strain ratio of approximately 5.0. A visual inspection of Figure 6 indicates that, over the apparent brittle failure range, a linear correlation would fit the data rather well.

Granite Data

Correlation. Figure 7 shows a plot of normalized axial strain, ϵ/ϵ_{uc} , vs normalized maximum shear stress, $(\sigma_1 - \sigma_3)/2\sigma_{uc}$, for the granite data given in Table A1 of Appendix A. The expression for the line best fitting all the data is as follows:

$$\frac{\epsilon}{\epsilon_{uc}} = 0.26 + 0.73 \left(\frac{\sigma_1 - \sigma_3}{\sigma_{uc}} \right) + 0.02 \left(\frac{\sigma_1 - \sigma_3}{\sigma_{uc}} \right)^2 \quad (20)$$

Data scatter. Like the sandstone data, data scatter for the granite tests was reasonable considering the sources of data and range of representative unconfined compressive strengths ($80 \text{ MPa (11,600 psi)} \leq \sigma_{uc} \leq 248 \text{ MPa (36,000 psi)}$). It is interesting to note that the Stone Mountain granite (and perhaps the Quartz Diorite data) data in Figure 7 deviate from the general trend. One possible explanation for this deviation might be that the loading frame used in these tests may have been insufficiently stiff for a high modulus rock like granite. A soft loading frame could cause dumping of strain energy stored in the loading frame resulting in a low observed deviator stress at failure.

Brittle-ductile transition. An inspection of available stress-strain curves for the granite data given in Table A1 indicated that all tests behaved in a brittle manner. It is interesting to note the almost linear nature of Equation 20.

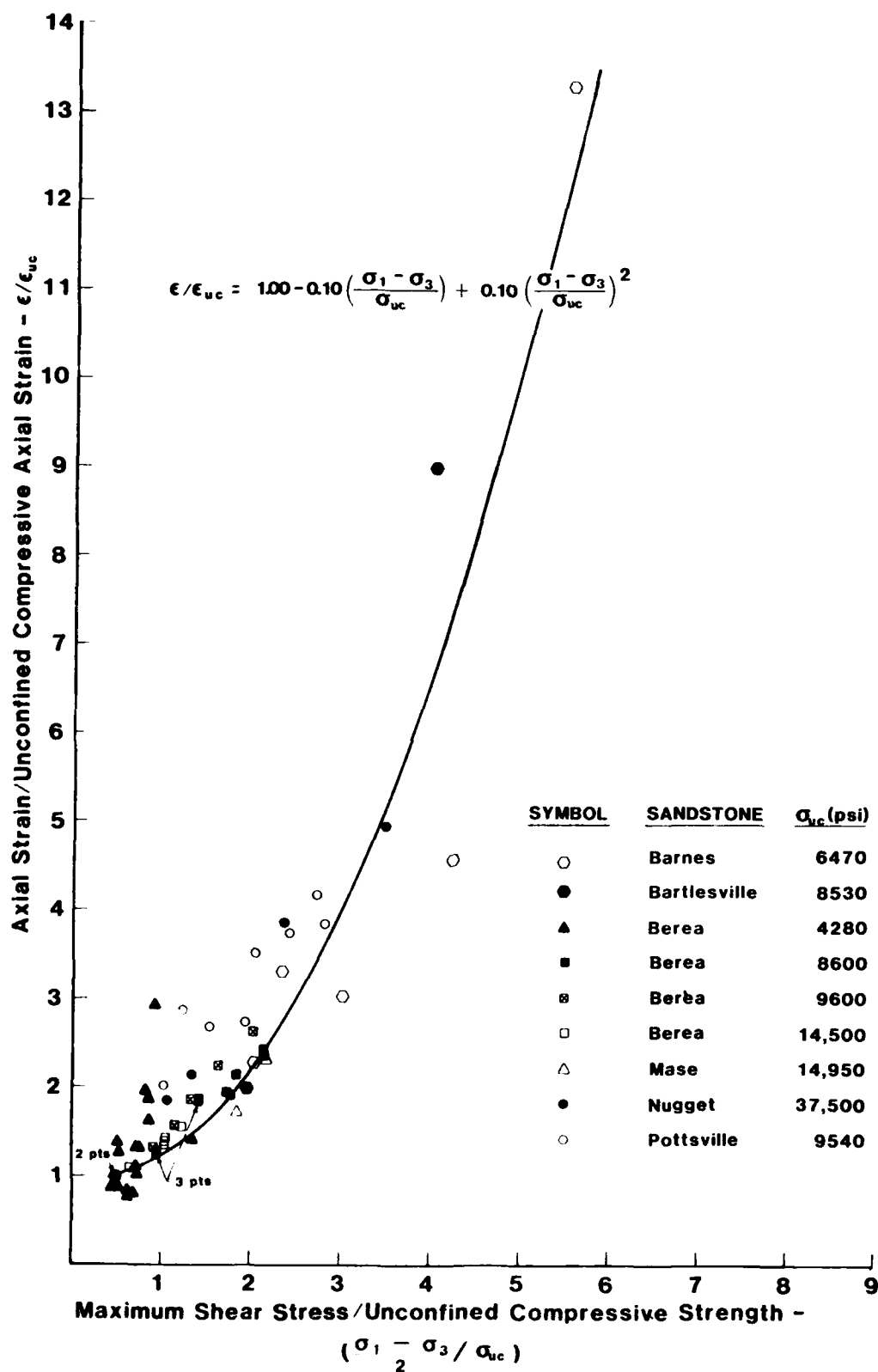


Figure 6. Plot of normalized axial strain versus normalized maximum shear stress for sandstone

Since $\sigma_3 = 0$, for the unconfined compression test the unconfined compressive stress σ_{uc} is indeed the major principal stress σ_1 (i.e. when $\sigma_3 = 0$ then $\sigma_1 = \sigma_{uc}$) for pure compression loading (i.e. no tensile stresses generated), the minimum value the normalized maximum shear stress component in Equation 15 can assume is:

$$\frac{\sigma_1}{2\sigma_{uc}} = \frac{\sigma}{2\sigma_{uc}} = 0.5 \quad (16)$$

Likewise, when $\sigma_3 = 0$ and $\epsilon = \epsilon_{uc}$, the minimum value of the dependent normalized strain component is:

$$\frac{\epsilon}{\epsilon_{uc}} = \frac{\epsilon_{uc}}{\epsilon_{uc}} = 1.0 \quad (17)$$

Equations obtained from Curve Fit required adjustments so that minimum values were satisfied. Putting Equations 16 and 17 into Equation 15 resulted in required expressions that assigned values of constants a , b and c must satisfy for the unconfined case as follows:

$$1.0 = a + 0.5b + 0.25c \quad (18)$$

Curve Fit values of a , b , and c were then adjusted so as to satisfy Equation 18 and provide a visual best fit of the data.

Sandstone Data

Correlation. Figure 6 shows a plot of normalized axial strain, ϵ/ϵ_{uc} vs normalized maximum shear stress, $(\sigma_1 - \sigma_3)/2\sigma_{uc}$, for the sandstone data given in Table A1 of Appendix A. The expression for the line in Figure 6 is as follows:

$$\frac{\epsilon}{\epsilon_{uc}} = 1.00 - 0.10 \left(\frac{\sigma_1 - \sigma_3}{\sigma_{uc}} \right) + 0.10 \left(\frac{\sigma_1 - \sigma_3}{\sigma_{uc}} \right)^2 \quad (19)$$

Forcing the curve to satisfy the minimum requirements for the maximum shear stress and strain components $((\sigma_1 - \sigma_3)/2\sigma_{uc}$ and ϵ/ϵ_{uc} , respectively) as discussed above shifted the curve slightly to the right as evident in Figure 6.

Data scatter Considering the sources of data and the range of strengths as reflected by the unconfined compressive strengths listed in Figure 6 ($30 \text{ MPa (4280 psi)} \leq \sigma_{uc} \leq 260 \text{ MPa (37,500 psi)}$), data scatter is not thought to be excessive. It should also be noted that the sandstone data include tests on wet and dry specimens as well as drained and undrained tests. Robinson (1955), Byerlee and Brace (1967), Byerlee (1975) and others have demonstrated the validity of effective stress in porous rocks such as sandstone. Most rocks

the literature by projecting from the point of failure to the appropriate axis defining stress and strain values. Values assigned for stress and strain are subject to some error due to the inaccuracies of scaling the data. The data represent results from a variety of different testing devices and, no doubt, testing procedures. Nicholson (1983) is of the opinion that test results can vary significantly for tests conducted in different triaxial devices even though care is taken to insure that test procedures, specimen preparation, and specimens are identical in so far as possible. Hence, considering the variety of data sources, one would expect a certain amount of data scatter.

During the course of this study two correlations between stress and strain were found to be of potential use for analyzing underground excavation behavior. In particular, these correlations were between the maximum shear stress $((\sigma_1 - \sigma_3)/2)$ and axial strain (ϵ) and between the minor principal stress (confining stress σ_3) and axial strain. Both stress parameters for each rock type were normalized with respect to the unconfined compressive strength for that rock type. Likewise, axial strains for each rock type were normalized with respect to the axial strain of the unconfined compressive strength for that rock type. Normalization allowed a relative comparison between stress-strain responses of specimen of the same rock type (i.e. sandstone and granite) but different stress-strain behavior. Table A1 (Appendix A) lists the test data according to rock type, principal stresses, axial strain, and symbols used in following correlation plots.

Correlations Between Maximum Shear Stress and Axial Strain

Curve Fitting

Considering the propensity for data scatter, a reasonable correlation was found to exist between normalized axial strain and normalized maximum shear stress for both the sandstone and granite data. Lines best fitting the data were obtained from a curve fitting routine, entitled Curve Fit, available on the Waterways Experiment Station's (WES) Honeywell DPS8 computer system. While several algebraic equation forms fit the data reasonably well, a second order polynomial resulted in the best correlation coefficient for both rock types. The general equation form may be expressed as:

$$\frac{\epsilon}{\epsilon_{uc}} = a + b \left(\frac{\sigma_1 - \sigma_3}{2 \sigma_{uc}} \right) + c \left(\frac{\sigma_1 - \sigma_3}{2 \sigma_{uc}} \right)^2 \quad (15)$$

Corresponding to the peak deviator stress there exists a unique value of axial strain (major principal strain) such that:

$$\epsilon_1 = \Delta L / L \quad (13)$$

$$\epsilon_{uc} = \Delta L_{uc} / L_{uc} \quad (14)$$

where

ϵ_1 = axial strain of triaxial compression specimen at failure

ΔL = total axial deformation of triaxial compression specimen at failure

L = original length of triaxial compression specimen at failure

ϵ_{uc} = axial strain of unconfined compression specimen at failure

ΔL_{uc} = total axial deformation of unconfined compression specimen at failure

L_{uc} = original length of unconfined compression specimen at failure

Note: subscripts denoting failure have been deleted to simplify presentation of terms.

In general $\epsilon_1 > \epsilon_{uc}$. For the unconfined compression case where $\sigma_3 = 0$ the deviator stress $\sigma_d = \sigma_1 = \sigma_{uc}$.

Figure 5 shows a Mohr's stress circle plot intended to be representative of the idealized stress-strain response at failure shown in Figure 4. The definition of the pertinent parameters are as defined in Figure 5. For a given rock material there exists a unique value of ϵ_{uc} (unconfined compression) and ϵ_1 (triaxial compression) for each stress circle. In developing the proposed empirical strain based failure criterion presented herein, the author assumed that the failure process is controlled by the major and minor principal stresses σ_1 and σ_3 , and that the intermediate principal σ_2 has no significant influence upon this process. Although this assumption oversimplifies actual rock failure behavior, there appears to be sufficient evidence, for example Jaeger and Cook (1969), to suggest that the influence of the intermediate principal stress can be ignored without introducing unacceptably large errors for practical problems

Triaxial Test Data

The author analyzed published data from approximately ninety triaxial tests on intact rock specimens. A total of 63 sandstone and 23 granite tests were analyzed. To be of use in this study the data had to include stress-strain response information. This information usually consisted of stress-strain curves. Data were taken from the stress-strain curves reported in

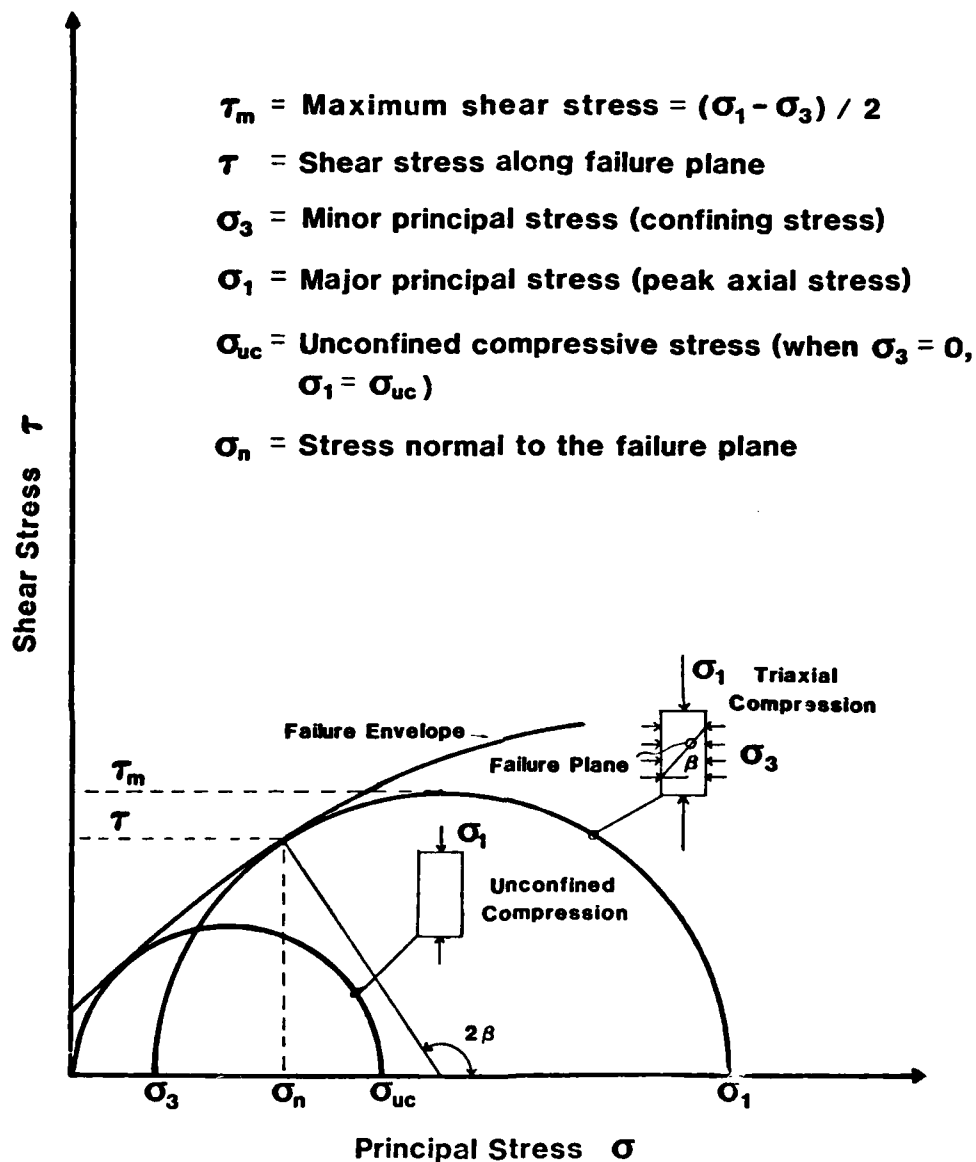


Figure 5. Idealized Mohr's stress circles and failure envelope for triaxial and unconfined compression tests with parameter definitions

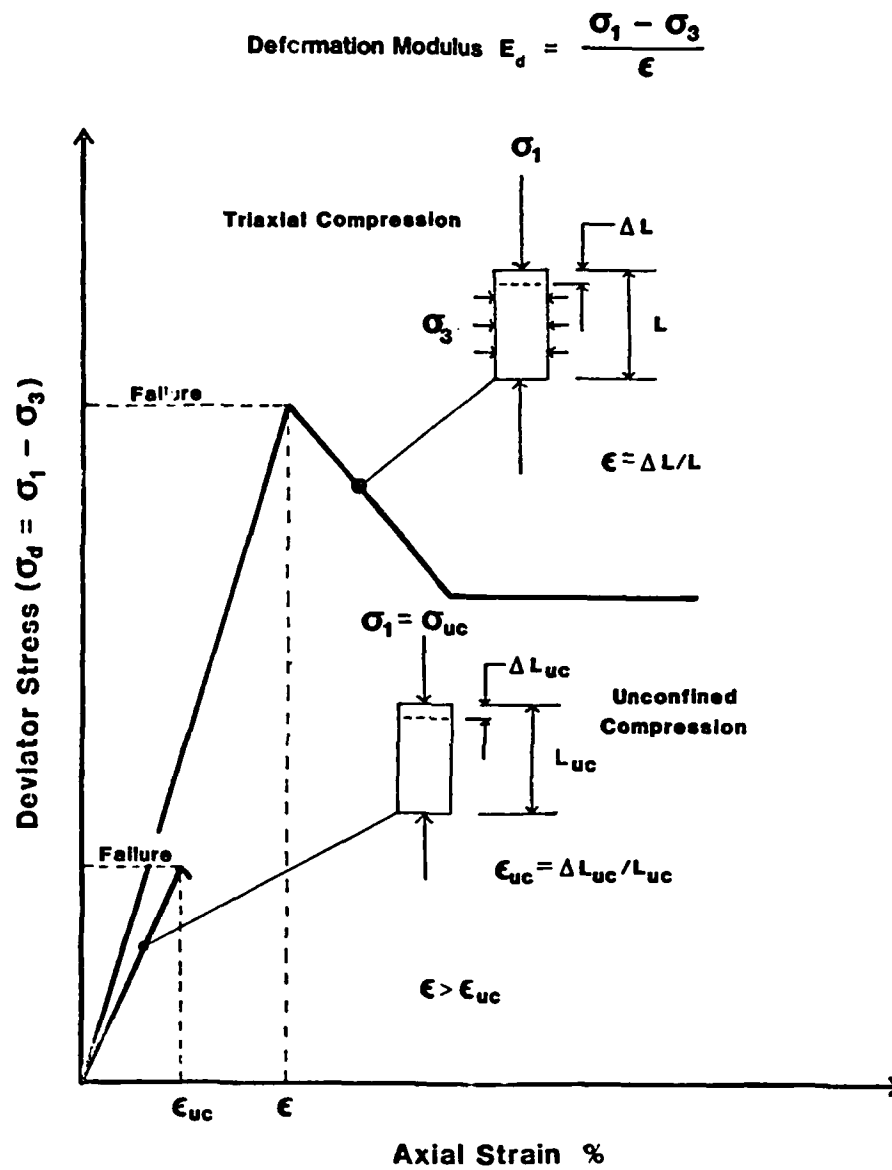


Figure 4. Idealized stress-strain response for triaxial and unconfined compression tests with parameter definitions

PART III: PROPOSED STRAIN BASED FAILURE CRITERION

Background

The simplest form of a strain based failure criterion would relate the major principal strain at failure to the principal stresses causing failure for a given intact rock type. If such an algebraic expression could be established for intact rock, then with suitable case history comparisons between predicted and observed behavior, the expression could be adjusted for rock mass. This part of the report will examine the feasibility of developing a strain based criterion for intact rock.

Specifically, empirical curves best defining stress-strain response at peak stress will be fitted to triaxial test data for two intact rock types - sandstone and granite. Intact rock was chosen for this study because of the availability of triaxial test data in the literature and because failure mechanisms for intact rock are simpler than mechanisms associated with single plane or rock mass failure.

Attributes

At the onset of this study it was decided that any proposed empirical criterion should possess four basic attributes:

1. The criterion should be simple and easy to understand.
2. Each parameter should be widely accepted by the rock mechanics practitioner.
3. Each parameter should be easy to measure with relatively cheap and repeatable tests.
4. The criterion should be functional in that all parameters are relevant to the analyses of underground excavation behavior.

With these attributes in mind a number of unsuccessful correlation attempts were made before a reasonable correlation was obtained between stress and strain. The scope of this report will not permit a discussion of the unsuccessful attempts. Hence, only those considerations leading to a reasonable correlation will be discussed herein.

Parameter Definitions

Figures 4 and 5 define the necessary parameters used in the development of the strain based criterion proposed herein. Figure 4 shows an idealized stress-strain response for simple triaxial compression and unconfined compression tests. Failure is defined by peak deviator stress ($\sigma_d = (\sigma_1 - \sigma_3)_{\max}$).

Limitations. The empirical rock mass failure criterion proposed by Hoek and Brown (1980) gives an approximate method for estimating the strength of jointed rock masses in terms of principal stresses. The method involves estimating the values of the empirical constants "m" and "s" from a description of the rock mass (summarized in Figure 3 and Table 3). These estimates, together with an estimate of the uniaxial compressive strength of the intact pieces of rock, can then be used to estimate the peak deviator stress σ_d for the jointed rock mass.

Hoek (1983) states that - "experience in using the values (of m and s) listed in Table 3 for practical engineering design suggests that they are somewhat conservative." In addition to being somewhat conservative, the criterion is valid only for brittle failure. Schwartz (1964) indicated that for Indiana Limestone a transition from brittle to ductile behaviour appeared to occur at a principal stress ratio of approximately $\sigma_1/\sigma_3 = 4.3$. Mogi (1966) concluded that the transition for most intact rock occurs at an average principal stress ratio $\sigma_1/\sigma_3 = 3.4$. Considering the curve fitting procedures and the variety of interpretations employed in defining the transition Hoek (1983) suggested a rough conservative rule-of-thumb to define the upper limits of stress for brittle failure of intact rock. That rule states that the minor principal (confining) stress must always be less than the uniaxial compressive strength of the material. While suitable guidance for rock mass behavior is not available it seems reasonable to conclude that brittle-ductile transitions for a rock mass would be of the same order of magnitude as for intact rock.

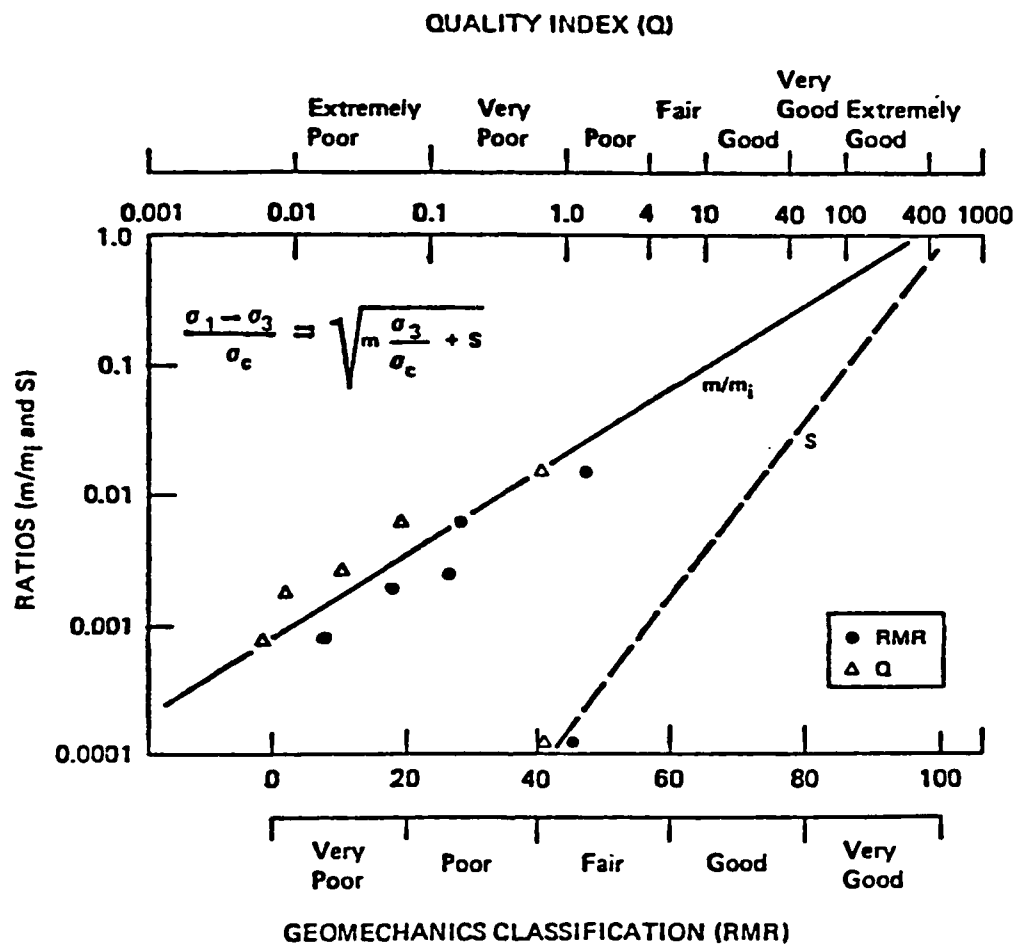


Figure 3. Criterion for in situ strength of rock
(after Hoek and Brown 1980)

form of the criterion applicable to the proposed constitutive model. Hence, single discontinuities will not be addressed herein.

"m" and "s" for rock mass. The presence of one or more sets of discontinuities in a rock mass will cause a reduction in the values of both "m" and "s". Unlike intact rock, relatively few sets of reliable triaxial test data for jointed rock are available. As such, the choice of "m" and "s" for a given rock mass must be based on the limited data available as well as back-analysis of documented cases of rock mass failures (as they become available) and a good measure of judgement. The only known set of triaxial test data available for undisturbed naturally jointed rock was that obtained by Jaeger (1970) for the Panguna andesite from the Bougainville open pit mine, Papua-New Guinea.

Figure 3 shows plots on the constant "s" and the ratio of "m/m_i," in which "m_i" is the value of "m" for intact rock material, against the Rock Mass Rating (RMR) system developed by Bieniawski (1973) and the Q-System developed by Barton, Lien and Lunde (1974). The RMR and Q classification ratings are estimates for the various categories of the Panguna andesite listed in Table 2. An inspection of Table 2 indicates that 5 data points for each classification rating are available for "m" with none above an RMR of approximately 46. One point for each classification rating (at an RMR of approximately 46) is available for "s" (plus s = 1.0 for intact rock). Straight lines were drawn on Figure 3 to give approximate relationships between "s" and "m/m_i" and the classification rating. Table 3 lists approximate relationships between rock mass quality and the material constants suggest by Hoek and Brown (1980) and Hoek (1983).

Priest and Brown (1983) recently proposed a probabilistic method for analysing the stability of slopes excavated in fractured, variable rock masses. The method is based on a Monte Carlo simulation routine with the use of the Janbu method of slices for which strength parameters are derived by the Hoek and Brown failure criterion. Priest and Brown (1983) used mathematical expressions to determine values of the material constants "m" and "s" from Figure 3 as follows:

$$R = \exp \frac{RMR - 95}{13.4} \quad (10)$$

$$m = R m_i \quad (11)$$

$$s = \exp \frac{RMR - 100}{6.3} \quad (12)$$

Where all parameters are defined above.

nonlinear failure envelope predicted by classical Griffith crack theory for plane compression (Hoek 1968) and by using a process of trial and error. The failure criterion is expressed by the following empirical relationship between the principal stresses at failure:

$$\sigma_1 / \sigma_{uc} = \sigma_3 / \sigma_{uc} + (m \sigma_3 / \sigma_{uc} + s)^{1/2} \quad (7)$$

Where

σ_1 = the major principal stress at failure

σ_3 = the minor principal stress at failure

σ_{uc} = the uniaxial compressive strength of the intact rock material

m & s = material constants that depend on the properties of the rock and on the extent to which it had been broken before being subjected to the failure stress σ_1 and σ_3 .

Equation 7 can be rewritten as follows:

$$\sigma_1 = \sigma_3 + (m \sigma_{uc} \sigma_3 + s \sigma_{uc}^2)^{1/2} \quad (8)$$

The scope of this report will not permit a detailed discussion of the criterion development. For details, the reader is referred to Hoek and Brown (1980) and Hoek (1983). However, for continuity of following discussions material constants "m" and "s" will be briefly discussed.

"m" and "s" for intact rock. For isotropic intact rock, the material constant "s" is defined as equal to 1.0. Hence, for intact rock Equation 8 becomes:

$$\sigma_1 = \sigma_3 + (m \sigma_{uc} \sigma_3 + 1.0 \sigma_{uc}^2)^{1/2} \quad (9)$$

A regression analysis was then used to determine the value "m" giving the best fit of Equation 9 to sets of normalized triaxial data (e.g. plots of σ_1 / σ_{uc} versus σ_3 / σ_{uc}) for a particular rock type. Hoek (1983) summarizes the best fit procedure. The value of "m" for a given rock type is dependent on the crystal fabric of the rock material and varies from approximately 5 to 30 (Hoek and Brown 1980).

"m" and "s" for discontinuous rock. The material constants "m" and "s" for single discontinuous planes are not constant as they are for intact rock, but vary with the orientation of the plane of weakness. It is unlikely that the criterion would be particularly useful in practice because of the somewhat complicated procedures required to define "m" and "s" nor is the single plane

cannot be expected to be representative of modulus defined at peak stress (i.e. slope of line OB in Figure 2). Finally, the assignment of a unique value for the deformation modulus ignores the fact that modulus varies with confining stress (minor principal stress) as indicated in Figure 1.

Proposed Constitutive Model

Minimum Requirements

A completely general and ideal constitutive model would consist of an algebraic expression, or set of algebraic expressions, that defines the complete rock mass stress-strain curve (to include prefailure, failure and postfailure). As a minimum, a model convenient for underground excavations would need to be a function of the major principal strain, the major and minor principal stress, and the rock mass conditions. Development of such a model would be a formidable, if not impossible, task. Nevertheless, the practicing geotechnical engineer is constantly searching for improved techniques for estimating stress-strain response.

Previous discussions on failure criteria indicated that a number of criteria are available for estimating stress at failure. However, no suitable criteria are available for estimating strain at failure. The development of a strain based failure criterion would provide a set of algebraic expressions for estimating deformation modulus up to failure as defined by peak deviator stress and strain at failure (i.e. Equation 4). The general form of expressions for peak deviator stress and strain at failure would be as follows:

$$\sigma_{d_f} = f(\sigma_1, \sigma_3, \text{rock mass conditions}) \quad (5)$$

$$\epsilon_{d_f} = f(\sigma_1, \sigma_3, \text{rock mass conditions}) \quad (6)$$

While such a formulation (Equation 4) would not be ideal in that, if required, postfailure stress-strain behavior would still have to be defined by some other model, it would provide a significant advancement for estimating the initial moduli values.

While no suitable strain based criteria existed for rock materials, a stress based criterion applicable to intact rock, discontinuous rock and rock mass is available. This stress based criterion, proposed by Hoek and Brown (1980), meets the minimum requirements as specified in Equation 5.

Hoek and Brown Failure Criterion

The Hoek and Brown failure criterion was developed by analogy with the

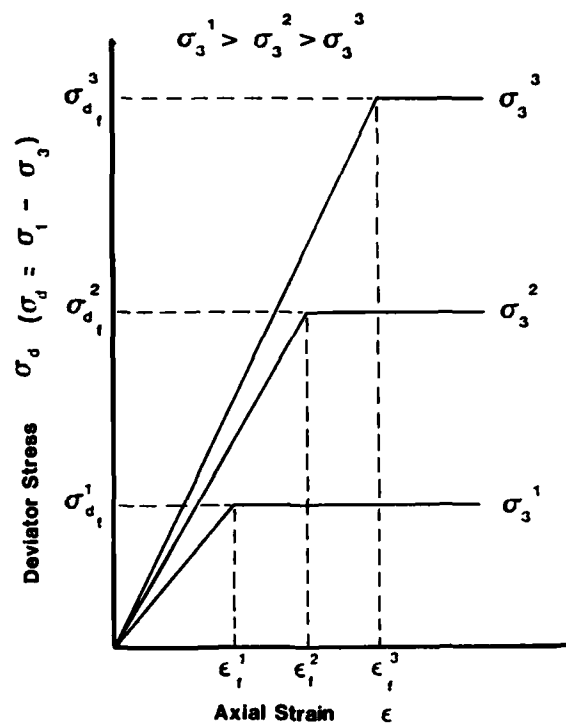


Figure 1. Idealized stress-strain responses for rock with varying minor principal stresses

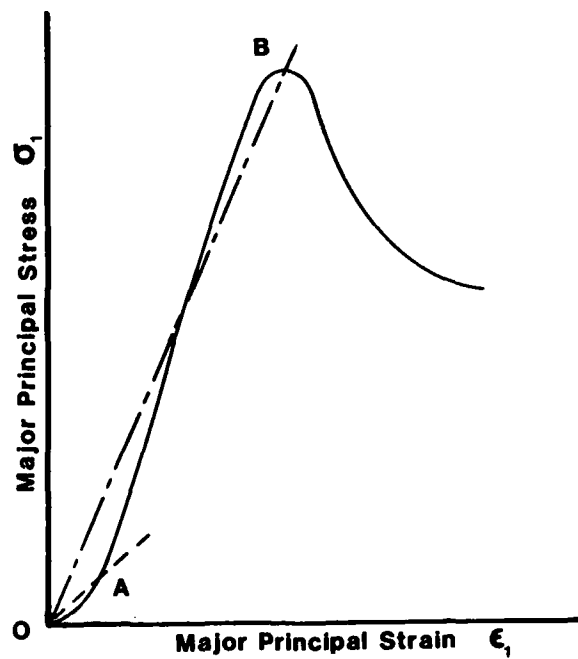


Figure 2. Hypothetical stress-strain curve typical of rock mass

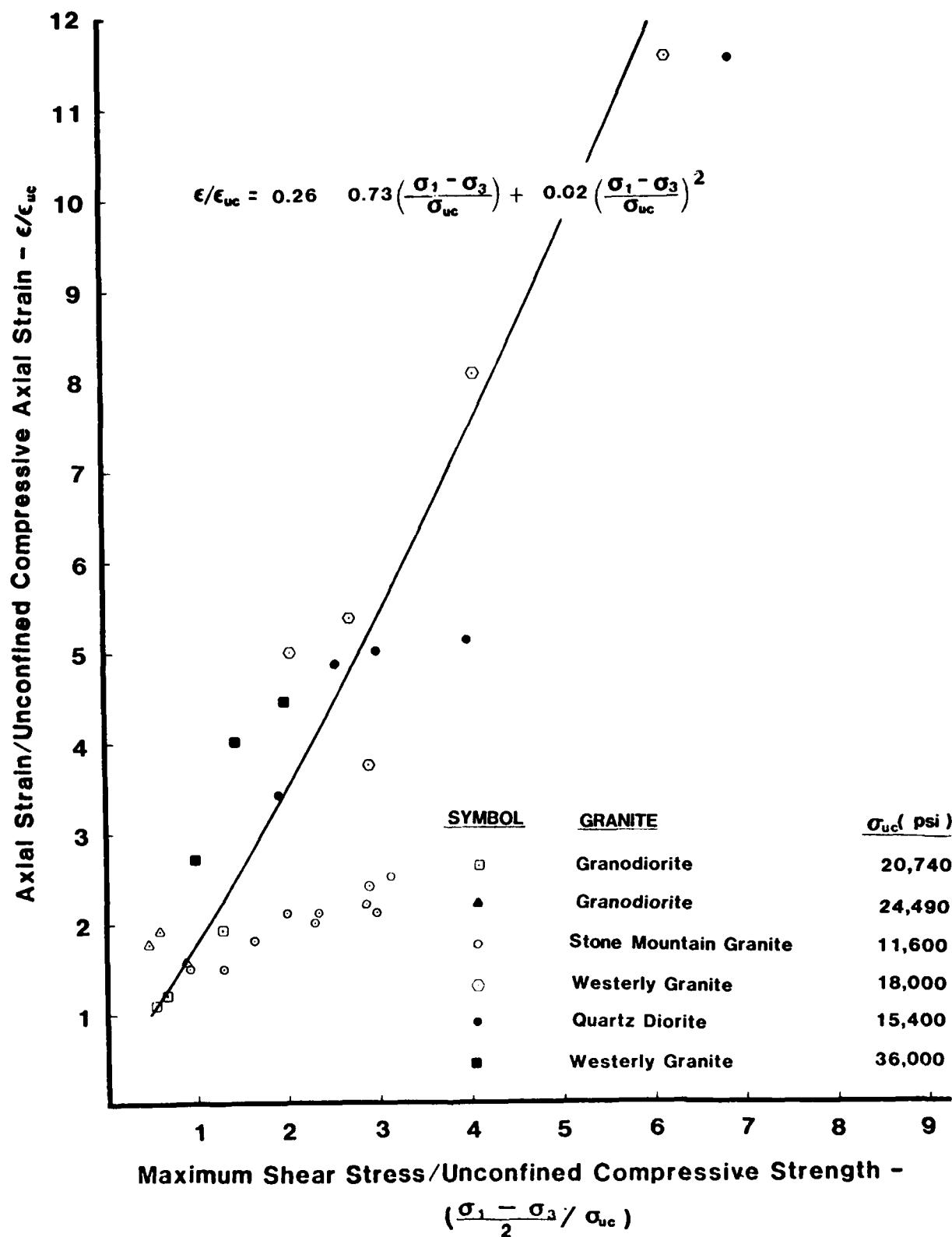


Figure 7. Plot of normalized axial strain versus normalized maximum shear stress for granite

Correlations Between Minor Principal Stress (Confining Stress) and Axial Strain

Curve Fitting

Curve fitting procedures for the normalized minor principal stress (σ_3/σ_{uc}) and normalized axial strain (ϵ/ϵ_{uc}) data were similar to those described above. The data are given in Table A1 of Appendix A. The Curve Fit program analysis resulted in a best fit second order polynomial of the general form:

$$\frac{\epsilon}{\epsilon_{uc}} = a + b \frac{\sigma_3}{\sigma_{uc}} + c \left(\frac{\sigma_3}{\sigma_{uc}} \right)^2 \quad (21)$$

For the unconfined compression test, where $\sigma_3 = \sigma_1$, Equation 17 is valid. Hence, from Equations 17 and 21, the minimum value of the normalized axial strain parameter is as follows:

$$\frac{\epsilon}{\epsilon_{uc}} = \frac{\epsilon_{uc}}{\epsilon_{uc}} = a = 1.0 \quad (22)$$

Values of the constants b and c in Equation 21 were then adjusted by trial and error to provide a visual best fit of the data and satisfy the minimum value criterion defined in Equation 22.

Sandstone Data

Figure 8 shows a plot of normalized axial strain, ϵ/ϵ_{uc} , vs normalized minor principal stress, σ_3/σ_{uc} , for the sandstone data given in Table A1 of Appendix A. The expression for the line best fitting all the data is as follows:

$$\frac{\epsilon}{\epsilon_{uc}} = 1.00 + \frac{\sigma_3}{\sigma_{uc}} (1.20 + 0.25 \frac{\sigma_3}{\sigma_{uc}}) \quad (23)$$

Granite Data

Figure 9 shows a plot of normalized axial strain, ϵ/ϵ_{uc} , vs normalized minor principal stress, σ_3/σ_{uc} , for the granite data given in Table A1 of Appendix A. The expression for the line best fitting all the data is as follows:

$$\frac{\epsilon}{\epsilon_{uc}} = 1.00 + \frac{\sigma_3}{\sigma_{uc}} \left(4.3 - 0.3 \frac{\sigma_3}{\sigma_{uc}} \right) \quad (24)$$

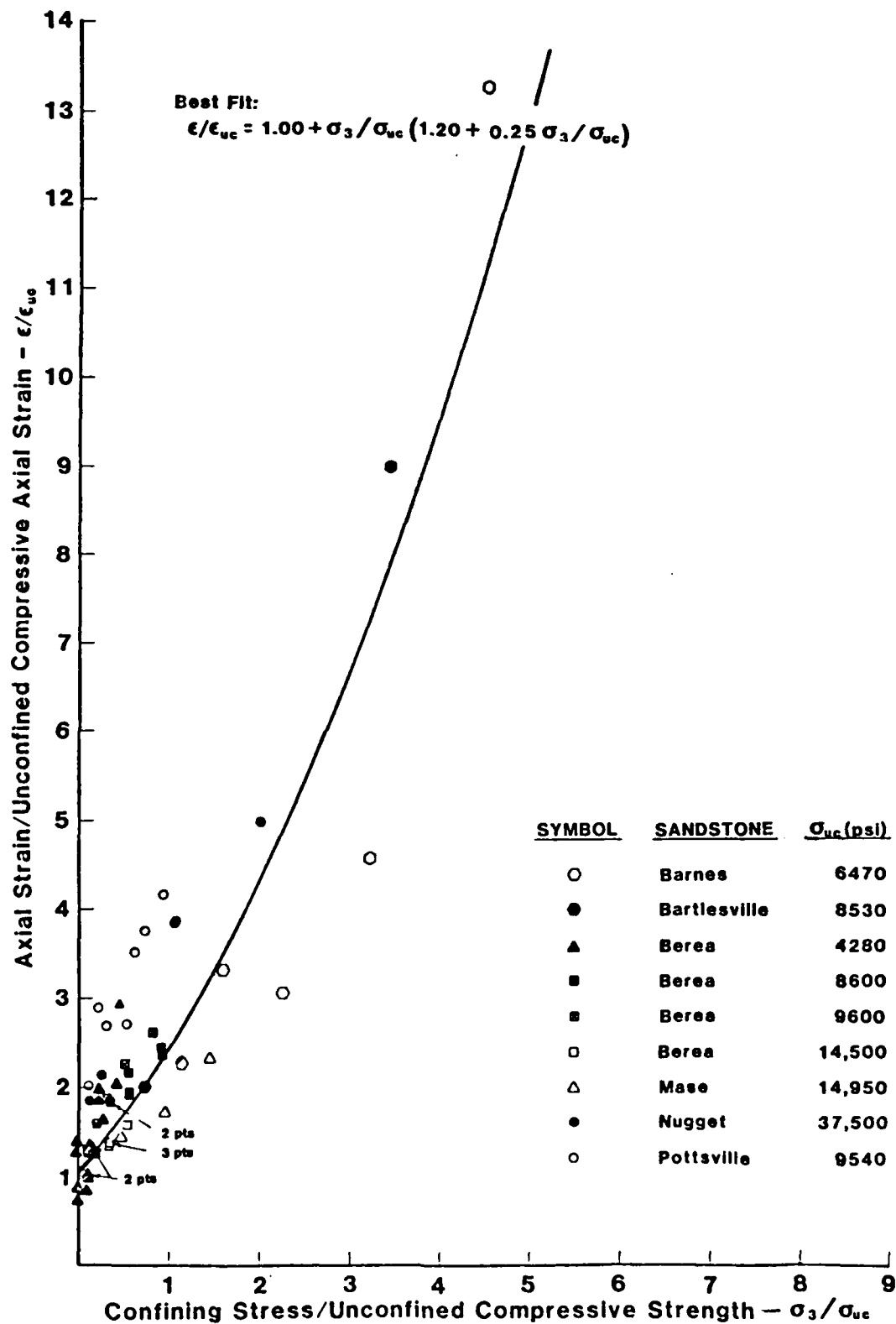


Figure 8. Plot of normalized axial strain versus normalized minor principal stress for sandstone

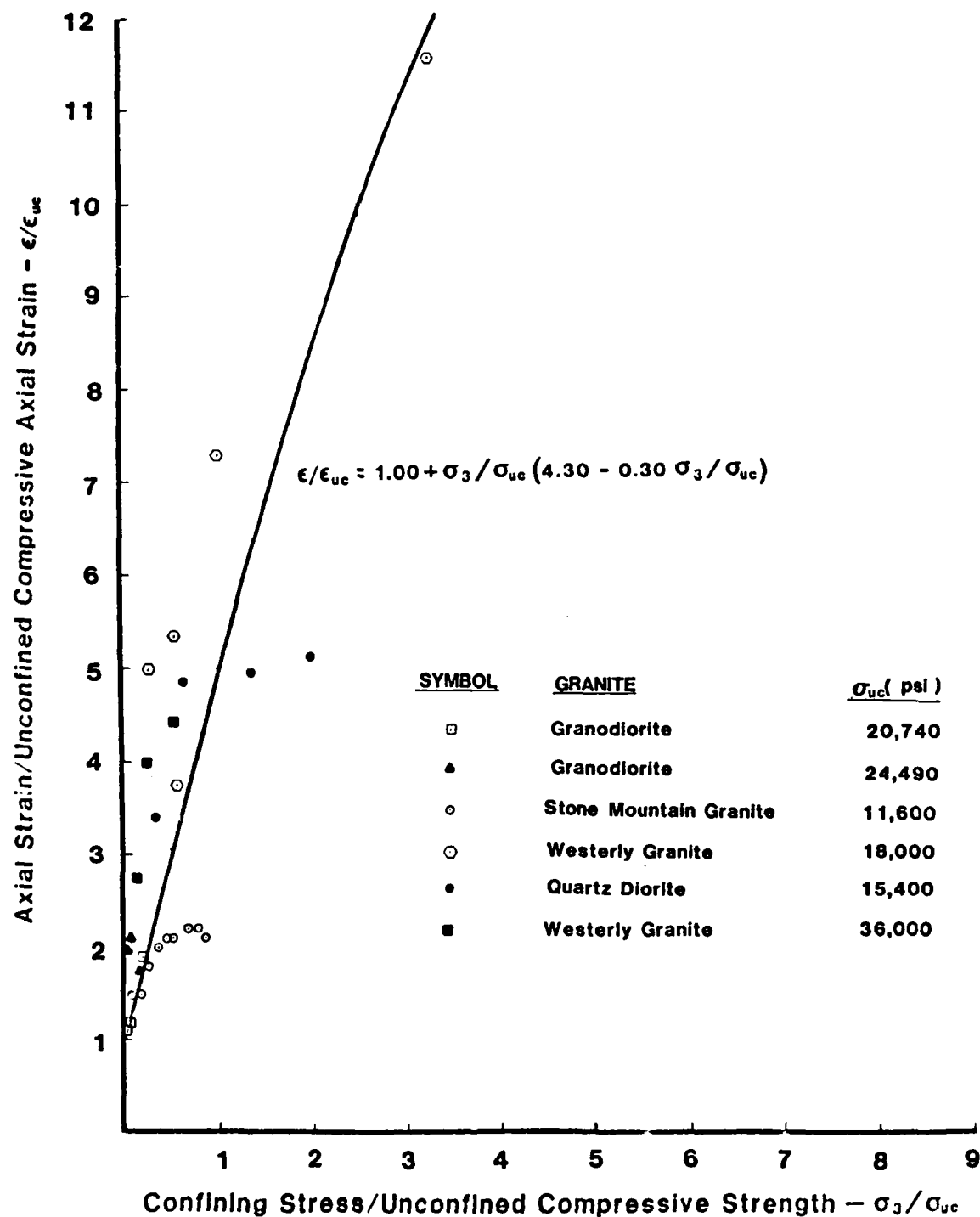


Figure 9. Plot of normalized axial strain versus normalized minor principal stress for granite

Deformation Modulus

The deformation modulus as defined by Equation 4 can be rewritten as follows:

$$\epsilon_d = \frac{\sigma_1 - \sigma_3}{E} \quad (25)$$

Hence, in order to predict deformation modulus one must be able to predict the principal stresses at failure and axial strain at failure. Hoek and Brown's (1980) failure criterion, as defined in Equation 7, offers a viable means of predicting the normalized principal stress difference at failure. For convenience Equation 7 can be expressed as:

$$\frac{\sigma_1 - \sigma_3}{\sigma_{uc}} = (m \sigma_3 / \sigma_{uc} + s)^{1/2} \quad (26)$$

Strain based criterion proposed herein allows estimates of strain at failure for intact sandstone and granite. As proposed, normalized axial strain at failure is expressed as functions of either normalized maximum shear stress or normalized minor principal stress. Strain expressions in terms of the minor principal stress represent the more convenient form for two reasons. First, note that the Hoek and Brown criterion, Equation 26, expresses the normalized deviator stress as a function of the normalized minor principal stress (as well as the constants m and s).

Expressing the normalized strain as a function of normalized minor principal stress would reduce the number of stress parameters in Equation 25 to one; that being the minor principal stress. Finally, in underground excavations, the minor principal stress is known or assumed to within reasonable tolerances at two boundaries; at the excavation surface and at some unknown distance within the wall rock unaffected by the relaxation process. The major principal stress, however, is seldom known except at some unspecified distance within the wall rock. Numerical analyses require a known boundary condition, such as at the excavation surface where both the location of the boundary and minor principal stresses are known. Hence, deformation moduli expressed in terms of the minor principal stress are more conducive to numerical modeling.

A dimensionless form of the deformation moduli expressed as a function of the minor principal stress can be obtained for intact sandstone or granite by substituting the left hand sides of Equation 26 and Equations 23 or 24 (for sandstone or granite respectively) into Equation 25 as follows:

$$E_d = \frac{\sigma_1 - \sigma_3}{\sigma_{uc}} / \frac{\epsilon}{\epsilon_{uc}} \quad (27)$$

or rewritten in terms of the normalized minor principal stress ratio the Equation for sandstone becomes:

$$\frac{E_d \epsilon_{uc}}{\sigma_{uc}} = \sqrt{\frac{15.0 \frac{\sigma_3}{\sigma_{uc}} + 1.0}{1.0 + \frac{\sigma_3}{\sigma_{uc}} (1.2 + 0.25 \frac{\sigma_3}{\sigma_{uc}})}} \quad (28)$$

where $m = 15$ (from Table 3)

and for granite

$$\frac{E_d \epsilon_{uc}}{\sigma_{uc}} = \sqrt{\frac{25.0 \frac{\sigma_3}{\sigma_{uc}} + 1.0}{1.0 + \frac{\sigma_3}{\sigma_{uc}} (4.3 - 0.3 \frac{\sigma_3}{\sigma_{uc}})}} \quad (29)$$

where $m = 25$ (from Table 3)

Equations 28 and 29 are plotted in Figure 10. It is interesting to note that for small increases in the σ_3/σ_{uc} ratio both rock types exhibit sharp increases in the dimensionless $E_d \epsilon_{uc}/\sigma_{uc}$ ratio up to a clearly defined peak. Granite peaks at a smaller σ_3/σ_{uc} ratio ($\sigma_3/\sigma_{uc} = 0.2$) than sandstone ($\sigma_3/\sigma_{uc} = 0.5$). With increasing σ_3/σ_{uc} ratio past peak the $E_d \epsilon_{uc}/\sigma_{uc}$ ratio decreases. However, note that for σ_3/σ_{uc} ratios greater than approximately 1.0 Equation 29 for intact granite predicts a lower modulus that would be obtained from unconfined compression tests (i.e. for unconfined compression $E_d \epsilon_{uc}/\sigma_{uc} = 1.0$). Sandstone, on the other hand, requires a σ_3/σ_{uc} of greater than approximately 3.0 before the $E_d \epsilon_{uc}/\sigma_{uc}$ ratio falls below 1.0.

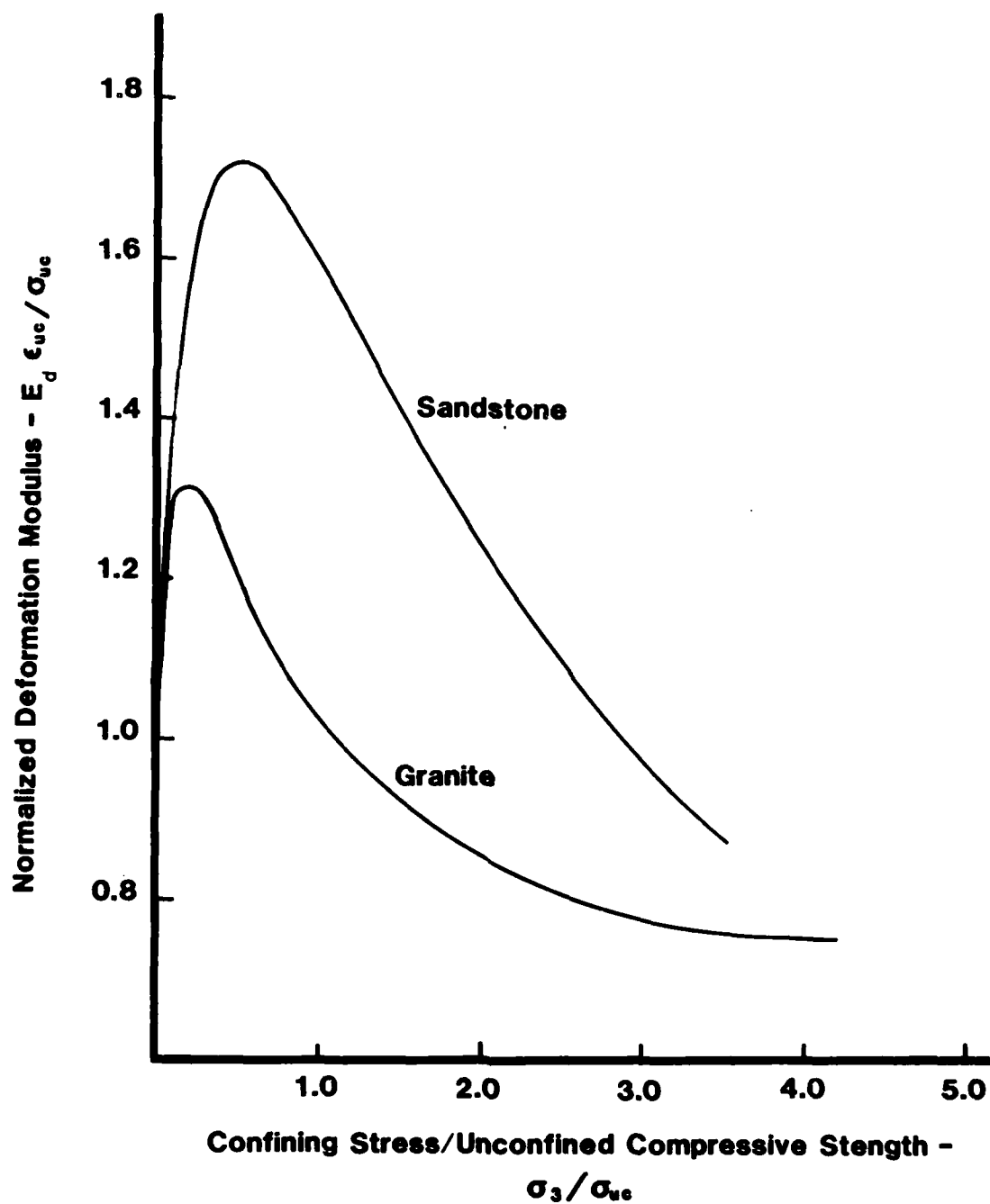


Figure 10. Plot of normalized deformation modulus versus normalized confining stress for sandstone and granite

PART IV: SUMMARY AND CONCLUSIONS

Summary

The constitutive model proposed in this report equates the deformation modulus to the ratio of the deviator stress vs axial strain at failure. For the model to be of practical significance requires failure criteria which define both the deviator stress at failure and strain at failure. The Hoek and Brown (1980) failure criterion offers a potentially powerful tool for defining the deviator stress in that the criterion expresses the deviator stress as a function of minor principal stress normalized with respect to the unconfined compressive strength and material constants "m" and "s." The material constants vary depending upon rock conditions ranging from intact to a highly jointed rock mass. Hoek and Brown (1980) offer the only criterion that addresses rock mass failure.

A detailed literature search failed to locate a criterion suitable for rock materials that defines strain at failure. In an effort to fill this void a feasibility study was conducted to evaluate the possibility of developing a strain based criterion for rock materials.

Stress-strain responses for two rock types, sandstone and granite, were evaluated. Reasonable correlations were found to exist between axial strain at failure normalized with respect to unconfined compressive axial strain at failure and the following:

1. Maximum shear stress normalized with respect to the unconfined compressive strength.
2. Minor principal stress (confining stress) normalized with respect to the unconfined compressive strength.

The Hoek and Brown criterion and the latter correlation provides an expression for estimating deformation modulus as a function of the minor principal stress.

Conclusions

This study has demonstrated the feasibility of developing empirical relationships defining strain at failure for rock materials, at least for intact sandstone and granite. Practical application requires that the relationships be extended to rock mass conditions. Extension of the criteria can be accomplished in two ways. The obvious approach would consist of a series of triaxial tests on specimens of sufficient size to be representative

of various rock mass conditions. Such an approach is prohibitively expensive and would require large diameter high load triaxial equipment not commonly available. A second alternative approach would consist of using numerical analysis of existing case histories to adjust the algebraic form of criteria so that predicted behavior matches observed behavior. Such an approach would unavoidably impose an additional assumption, that assumption being the validity of the numerical technique.

Work is in progress at the Waterways Experiment Station in an effort to develop a strain base failure criterion applicable to rock mass conditions. If successful, the criterion will significantly increase our ability to provide meaningful constitutive relationships between stress and strain necessary for rational design of underground excavations.

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Table 1
Major Failure Criteria for Rock Materials
(Bieniawski 1984)

Name of Criterion	Proposer	Date	Reference
Coulomb-Navier	Coulomb	1773	Jaeger and Cook 1979
Maximum shear stress	Tresca	1864	Nadai 1950
Maximum principal stress	Rankine	1869	Nadai 1950
Maximum elastic strain	St. Venant	1870	Nadai 1950
Constant elastic energy of deformation	Beltrami	1885	Nadai 1950
Shear failure	Mohr	1900	Jaeger and Cook 1979
Constant octahedral shearing stress	Huber Hencky	1904 1920	Jaeger and Cook 1979
Second invariant of stress deviation	von Mises	1913	Jaeger and Cook 1979
Griffith-original	Griffith	1921	Griffith 1921,1924
Statistical failure theory	Weibull	1939	Weibull 1952
Fracture toughness	Irwin	1960	Irwin 1960
Griffith-modified	McClintock-Walsh	1962	McClintock and Walsh 1962 Hoek and Bieniawski 1965
Griffith-extended	Murrell	1963	Murrell 1963
Empirical rock mass strength	Hoek-Brown	1980	Hoek and Brown 1980

Table 2
The Rock Mass Rating and Q-System Classifications of
Panguna Andesite Samples (Hoek and Brown 1980)

	Intact rock specimens	Undisturbed core samples	Recompacted, graded samples	Fresh to slightly weathered samples	Moderately weathered samples	Highly weathered samples
Ratio m/m_0	1.00	0.0147	0.0061	0.0021	0.0016	0.0006
Value of constant s	1.00	0.002	0	0	0	0
CSIR CLASSIFICATION	Intact strength rating	15	15	15	15	15
	RQD rating	20	3	3	3	3
	Joint spacing rating	30	5	5	5	5
	Joint condition rating	25	20	10	5	0
	Groundwater rating	10	8	5	5	5
	Joint orientation rating	0	-5	-10	-12	-12
	CSIR total rating	100	46	28	18	8
NGI CLASSIFICATION	RQD	100	10	10	10	10
	Joint set number J_n	1	9	12	18	20
	Joint roughness number J_r	4	3	1.5	1	1
	Joint alteration number J_a	0.75	2	4	6	8
	Joint water reduction J_w	1	1	1	1	1
	Stress reduction factor SRF	1	2.5	5	10	10
	NGI quality index Q	533	0.67	0.06	0.02	0.006

Table 3

Approximate Relationship Between Rock Mass Quality,
Rock Type, and Material Constants (Hoek 1983)

<p>Empirical failure criterion</p> $\sigma_1 = \sigma_3 + (m\sigma_c\sigma_3^2 + \sigma_c^2)^{1/2}$ <p>σ_1 = major principal stress σ_3 = minor principal stress σ_c = uniaxial compressive strength of intact rock m, s = empirical constants</p>	CARBONATE ROCKS WITH WELL DEVELOPED CRYSTAL CLEAVAGE dolomite, limestone and marble	LITHIFIED ARGILLACEOUS ROCKS mudstone, siltstone, shale and slate (tested normal to cleavage)	ARENACEOUS ROCKS WITH STRONG CRYSTALS AND POORLY DEVELOPED CRYSTAL CLEAVAGE sandstone and quartzite	FINE GRAINED POLYMINERALIC IGNEOUS CRYSTALLINE ROCKS andesite, diorite, diorite and rhyolite	COARSE GRAINED POLYMINERALIC IGNEOUS AND METAMORPHIC CRYSTALLINE ROCKS amphibolite, gabbro, gneiss, granite, norite and quartzdiorite
<p>INTACT ROCK SAMPLES</p> <p>Laboratory size samples free from pre-existing fractures</p> <p>Bieniawski (CSIR) rating 100 Barton (NGI) rating 500</p>	<p>$m = 7.0$ $s = 1.0$</p>	<p>$m = 10.0$ $s = 1.0$</p>	<p>$m = 15.0$ $s = 1.0$</p>	<p>$m = 17.0$ $s = 1.0$</p>	<p>$m = 25.0$ $s = 1.0$</p>
<p>VERY GOOD QUALITY ROCK MASS</p> <p>Tightly interlocking undisturbed rock with rough unweathered joints spaced at 1 to 3 m</p> <p>Bieniawski (CSIR) rating 85 Barton (NGI) rating 100</p>	<p>$m = 3.5$ $s = 0.1$</p>	<p>$m = 5.0$ $s = 0.1$</p>	<p>$m = 7.5$ $s = 0.1$</p>	<p>$m = 8.5$ $s = 0.1$</p>	<p>$m = 12.5$ $s = 0.1$</p>
<p>GOOD QUALITY ROCK MASS</p> <p>Fresh to slightly weathered rock, slightly disturbed with joints spaced at 1 to 3 m</p> <p>Bieniawski (CSIR) rating 65 Barton (NGI) rating 10</p>	<p>$m = 0.7$ $s = 0.004$</p>	<p>$m = 1.0$ $s = 0.004$</p>	<p>$m = 1.5$ $s = 0.004$</p>	<p>$m = 1.7$ $s = 0.004$</p>	<p>$m = 2.5$ $s = 0.004$</p>
<p>FAIR QUALITY ROCK MASS</p> <p>Several sets of moderately weathered joints spaced at 0.3 to 1 m, disturbed</p> <p>Bieniawski (CSIR) rating 44 Barton (NGI) rating 1</p>	<p>$m = 0.14$ $s = 0.0001$</p>	<p>$m = 0.20$ $s = 0.0001$</p>	<p>$m = 0.30$ $s = 0.0001$</p>	<p>$m = 0.34$ $s = 0.0001$</p>	<p>$m = 0.50$ $s = 0.0001$</p>
<p>POOR QUALITY ROCK MASS</p> <p>Numerous weathered joints at 30 to 500mm with some gouge. Clean, compacted rockfill</p> <p>Bieniawski (CSIR) rating 23 Barton (NGI) rating 0.1</p>	<p>$m = 0.04$ $s = 0.00001$</p>	<p>$m = 0.05$ $s = 0.00001$</p>	<p>$m = 0.08$ $s = 0.00001$</p>	<p>$m = 0.09$ $s = 0.00001$</p>	<p>$m = 0.13$ $s = 0.00001$</p>
<p>VERY POOR QUALITY ROCK MASS</p> <p>Numerous heavily weathered joints spaced at 50mm with gouge. Waste rock</p> <p>Bieniawski (CSIR) rating 3 Barton (NGI) rating 0.01</p>	<p>$m = 0.007$ $s = 0$</p>	<p>$m = 0.010$ $s = 0$</p>	<p>$m = 0.015$ $s = 0$</p>	<p>$m = 0.017$ $s = 0$</p>	<p>$m = 0.025$ $s = 0$</p>

APPENDIX A
SANDSTONE AND GRANITE TEST DATA

Table A1
Test Data for Sandstone and Granite

Type of Rock and symbol	σ_1 Ksi	σ_3 Ksi	σ_{uc} Ksi	ϵ %	ϵ_{uc} %	$\frac{\sigma_1 - \sigma_3}{2\sigma_{uc}}$	$\frac{\sigma_3}{\sigma_{uc}}$	$\frac{\epsilon}{\epsilon_{uc}}$	Reference
<u>Sandstone</u>									
○ Rarns	---	---	6.47	---	1.440	0.50	0.0	1.00	Handin & Hager (1957)
	33.83	7.36	---	3.280	---	2.05	1.14	2.28	
	54.43	14.71	---	4.400	---	3.07	2.27	3.06	
	28.69	7.36	---	3.200	---	2.34	1.62	3.33	
	53.10	14.71	---	4.400	---	4.22	3.23	4.58	
● Bartlesville	---	---	8.53	---	2.000	0.50	0.0	1.00	Waterways Ex- periment Station (1984)
	48.80	14.71	---	4.000	---	2.00	1.7	1.73	
	98.39	29.42	---	18.000	---	4.04	3.45	9.00	
	---	---	4.28	---	0.430	0.50	0.0	1.00	
	3.83	0.01	---	0.38	---	0.45	0.002	0.89	
▲ Berea	4.80	0.20	---	0.38	---	0.54	0.05	0.89	
	7.92	1.00	---	0.85	---	0.81	0.23	1.98	
	5.49	0.24	---	0.37	---	0.61	0.06	0.85	
	8.71	1.20	---	0.71	---	0.88	0.28	1.64	
	7.00	0.60	---	0.49	---	0.75	0.14	1.13	
	6.79	0.48	---	0.48	---	0.74	0.11	1.01	
	6.43	0.44	---	0.36	---	0.70	0.10	0.83	
	5.74	0.22	---	0.31	---	0.64	0.05	0.73	
	6.99	0.66	---	0.58	---	0.74	0.15	1.34	
	3.49	1.00	---	0.80	---	0.88	0.23	1.87	
	7.16	0.60	---	0.58	---	0.77	0.140	1.34	
	10.26	2.00	---	1.27	---	0.97	0.47	2.94	
	10.14	1.80	---	0.88	---	0.97	0.42	2.05	
	4.59	0.01	---	0.55	---	0.54	0.002	1.27	
	4.45	0.01	---	0.60	---	0.52	0.002	1.39	

(Continued)

(Sheet 1 of 4)

Table A1 (Continued)

Type of Rock and symbol	σ_1	σ_3	σ_{UC}	ϵ	ϵ_{UC}	$\frac{\sigma_1 - \sigma_3}{2\sigma_{UC}}$	$\frac{\sigma_3}{\sigma_{UC}}$	$\frac{\epsilon}{\epsilon_{UC}}$	Reference
■ Berea	---	---	8.60	---	0.59	0.50	0.0	1.00	Aldrich (1969)
	16.50	1.00	---	0.75	---	0.90	0.12	1.27	
	27.80	3.00	---	1.10	---	1.44	0.35	1.86	
	28.20	3.00	---	1.08	---	1.47	0.35	1.83	
	35.00	5.00	---	1.16	---	1.74	0.58	1.97	
	35.80	5.00	---	1.13	---	1.79	0.58	1.92	
	36.50	5.00	---	1.28	---	1.83	0.58	2.17	
	45.40	8.00	---	1.39	---	2.17	0.93	2.36	
	44.70	8.00	---	1.45	---	2.13	0.93	2.46	
	---	---	9.60	---	0.57	0.50	0.0	1.00	
☒ Berea	18.30	1.00	---	0.74	---	0.90	0.10	1.31	Bruhn (1972)
	24.40	2.00	---	0.90	---	1.17	0.21	1.59	
	28.70	3.00	---	1.06	---	1.34	0.31	1.87	
	36.80	5.00	---	1.28	---	1.66	0.52	2.27	
	47.00	8.00	---	1.48	---	2.03	0.83	2.67	
	---	---	14.50	---	0.92	0.50	0.0	1.00	
□ Rerea	20.40	1.00	---	1.02	---	0.67	0.07	1.11	Hoshino & Koide (1970)
	29.60	3.00	---	1.16	---	0.92	0.21	1.26	
	35.80	5.00	---	1.25	---	1.06	0.35	1.36	
	35.50	5.00	---	1.29	---	1.05	0.35	1.40	
	35.30	5.00	---	1.27	---	1.05	0.35	1.38	
	42.30	8.00	---	1.45	---	1.25	0.55	1.58	
	---	---	14.95	---	0.90	0.50	0.0	1.00	
△ Muse	48.93	7.26	---	1.30	---	1.38	0.49	1.44	Hoshino & Koide (1970)
	70.25	14.51	---	1.55	---	1.86	0.91	1.72	
	87.18	21.77	---	2.10	---	2.33	1.46	2.33	

(Continued)

(Sheet 2 of 4)

Table A1 (Continued)

Type of Rock and symbol	σ_1	σ_3	σ_{uc}	ϵ	ϵ_{uc}	$\frac{\sigma_1 - \sigma_3}{2\sigma_{uc}}$	$\frac{\sigma_3}{\sigma_{uc}}$	$\frac{\epsilon}{\epsilon_{uc}}$	Reference
● Nugget	---	---	37,500	---	0.76	0.50	0.0	1.00	Brown & Swanson (1971)
	86.00	5.00	---	1.40	---	1.08	0.13	1.85	
	109.00	10.00	---	1.60	---	1.32	0.27	2.12	
	220.00	41.00	---	2.92	---	2.39	1.09	3.86	
	343.00	80.00	---	3.76	---	3.51	2.13	4.97	
○ Pottsville	---	---	9.54	---	0.58	0.50	0.0	1.00	Schwartz (1964)
	20.85	1.00	---	1.18	---	1.04	0.11	2.03	
	25.23	2.00	---	1.68	---	1.22	0.21	2.89	
	32.07	3.00	---	1.56	---	1.52	0.31	2.69	
	41.60	5.00	---	1.58	---	1.92	0.52	2.72	
	45.08	6.00	---	2.04	---	2.05	0.63	3.52	
	53.15	7.00	---	2.18	---	2.42	0.73	3.76	
	60.85	9.00	---	2.42	---	2.72	0.94	4.17	
	63.54	10.00	---	2.23	---	2.81	1.05	3.84	
	---	---	---	---	---	---	---	---	
Granite ■ Westerly	---	---	36.00	---	0.45	0.50	0.0	1.0	Brown, Swanson, Wawersik (1972)
	77.00	5.00	---	1.23	---	1.00	0.14	2.73	
	114.00	10.00	---	1.83	---	1.44	0.28	4.07	
	164.00	20.0	---	2.00	---	2.00	0.56	4.44	
△ Granodiorite	---	---	24.49	---	0.28	0.5	0.0	1.1	Stowe, Ainsworth (1968)
	24.45	0.25	---	0.49	---	0.49	0.01	1.75	
	30.00	1.00	---	0.53	---	0.59	0.04	1.89	
	48.00	4.00	---	0.44	---	0.90	0.16	1.57	

(Continued)

(Sheet 3 of 4)

Table A1 (Concluded)

Type of Rock and symbol	σ_1	σ_3	σ_{uc}	ϵ	ϵ_{uc}	$\frac{\sigma_1 - \sigma_3}{2\sigma_{uc}}$	$\frac{\sigma_3}{\sigma_{uc}}$	$\frac{\epsilon}{\epsilon_{uc}}$	Reference
□ Granodiorite	---	---	20.74	---	0.22	0.5	0.0	1.00	R. L. Stone (1969)
	24.37	0.25	---	0.24	---	0.58	0.01	1.09	
	30.00	1.0	---	0.26	---	0.71	0.05	1.18	
	56.54	4.0	---	0.42	---	1.27	0.19	1.91	
○ Stone Mountain	---	---	11.60	---	1.0	0.5	0.0	1.0	A. Schwartz (1964)
	22.60	1.0	---	1.5	---	0.93	0.09	1.5	
	32.70	2.0	---	1.5	---	1.32	0.17	1.5	
	41.00	3.0	---	1.8	---	1.64	0.26	1.8	
	59.70	4.0	---	2.0	---	2.31	0.34	2.0	
	51.70	5.0	---	2.1	---	2.01	0.43	2.1	
	60.30	6.0	---	2.1	---	2.34	0.52	2.1	
	79.70	7.0	---	2.5	---	3.13	0.60	2.5	
	74.50	8.0	---	2.2	---	2.87	0.69	2.4	
	74.10	9.0	---	2.3	---	2.81	0.77	2.2	
	79.10	10.0	---	2.1	---	3.00	0.86	2.1	
○ Westerly	---	---	18.00	---	0.26	0.5	0.0	1.0	W. S. Brown, S. R. Swanson (1970)
	79.00	5.0	---	1.3	---	2.06	0.28	5.0	
	108.00	10.0	---	1.4	---	2.72	0.56	5.38	
	114.00	10.0	---	0.98	---	2.89	0.56	3.77	
	166.00	20.0	---	2.1	---	4.06	1.11	8.08	
	284.00	60.0	---	3.0	---	6.22	3.33	11.54	
● Quartz Diorite	---	---	15.40	---	0.39	0.5	0.0	1.0	W. S. Brown, S. R. Swanson (1971)
	64.50	5.0	---	1.33	---	1.93	0.32	3.41	
	89.00	10.0	---	1.90	---	2.56	0.65	4.87	
	116.00	21.0	---	1.93	---	3.08	1.36	4.95	
	155.00	31.0	---	2.0	---	4.03	2.01	5.13	
	280.00	67.0	---	4.5	---	6.92	4.35	11.54	

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